

NMR spectroscopy

- Absorption (or emission) spectroscopy, as IR or UV. Detects the absorption of radiofrequencies (electromagnetic radiation) by certain nuclei in a molecule.
- Only nuclei with **spin number** (I) $\neq 0$ can absorb/emit electromagnetic radiation.
 - Even atomic number & even mass $\Rightarrow I = 0$ (^{12}C , ^{16}O)
 - Even atomic mass & odd atomic number $\Rightarrow I =$ whole integer (^{14}N , ^2H , ^{10}B)
 - Odd atomic mass $\Rightarrow I =$ half integer (^1H , ^{13}C , ^{15}N , ^{31}P)
- The **spin states** of the nucleus (m) are **quantified**:

$$m = I, (I - 1), (I - 2), \dots, -I$$

- Properly, m is called the **magnetic quantum number**.

Background (continued)

- For ^1H , ^{13}C , ^{15}N , ^{31}P (biologically relevant nuclei) then:

$$m = 1/2, -1/2$$

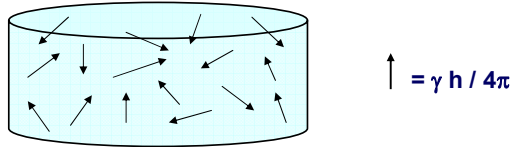
- This means that only two states (energy levels) can be taken by these nuclei.
- Another important parameter of each particular nuclei is the **magnetic moment** (μ), which can be expressed as:

$$\vec{\mu} = \gamma I \hbar / 2\pi$$

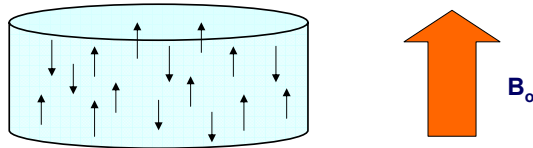
- It is a vector quantity that gives the direction and magnitude (or strength) of the 'nuclear magnet'
 - \hbar is the Planck constant
 - γ is the gyromagnetic ratio, and it depends on the nature of each nuclei.
- Different nuclei have different magnetic moments.

Effect of a magnetic field (for $I = 1/2$)

- In the ground state all nuclear spins are disordered, and there is no energy difference between them. They are **degenerate**:



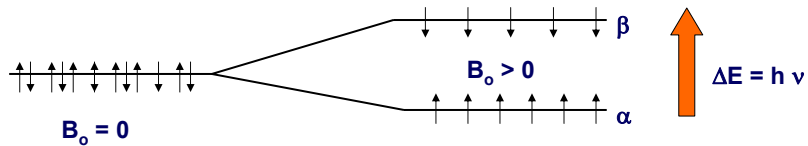
- Since they have a magnetic moment, when we apply a strong external magnetic field (B_0), they orient either against or with it:



- There is always a small excess of nuclei (**population excess**) aligned with the field than pointing against it.

Energy and populations

- Upon application of the external magnetic field we create an energy difference between nuclei aligned and against B_0 :



- Each level has a different **population** (N), and the difference between the two is related to the energy difference by the Boltzmann distribution:

$$N_\alpha / N_\beta = e^{\Delta E / kT}$$

- The ΔE for ^1H at 400 MHz ($B_0 = 9.5 \text{ T}$) is $3.8 \times 10^{-5} \text{ Kcal / mol}$

$$N_\alpha / N_\beta = 1.000064$$

- The surplus population is small when compared to UV or IR.

Energy and sensitivity

- The energy (for a single spin) is proportional to the magnetic moment of the nuclei and the external magnetic field:

$$\mathbf{E} = -\vec{\mu} \cdot \vec{B}_o \Rightarrow E_{(up)} = \gamma h B_o / 4\pi \text{ --- } E_{(down)} = -\gamma h B_o / 4\pi$$

$$\Delta E = \gamma h B_o / 2\pi$$

- This has implications on the energy (i.e., the intensity of the signal and sensitivity) that each nuclei can absorb:
 - Bigger magnets (bigger B_o) make more sensitive NMR instruments.
 - Nuclei with larger γ absorb/emit more energy and are therefore more sensitive. Sensitivity is proportional to μ , to $N_\alpha - N_\beta$, and to the 'coil magnetic flux', which are all dependent on γ . Therefore, it is proportional to γ^3 .

$$\gamma^{13C} = 6,728 \text{ rad / G}$$

1H is ~ 64 times more sensitive than ^{13}C just because of the γ

$$\gamma^{1H} = 26,753 \text{ rad / G}$$

- If we consider natural abundance, ^{13}C (~1%) ends up being 6400 times less sensitive...

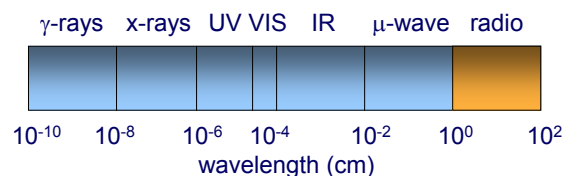
Energy and frequency

- Since energy is related to frequency, we can do some insightful math...

$$\Delta E = h \nu$$

$$\Delta E = \gamma h B_o / 2\pi \quad \left. \vphantom{\Delta E} \right\} \nu = \gamma B_o / 2\pi$$

- For 1H in normal magnets (2.35 - 18.6 T), this frequency is in the 100-800 MHz range. For ^{13}C , 1/4 of that...

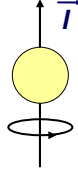


- To explain certain aspects of NMR, we need to refer to circular motion. Hz are not the best units to do so. We define the **precession** or **Larmor** frequency, ω :

$$\omega = 2\pi\nu \Rightarrow \omega_o = \gamma B_o \text{ (radians)}$$

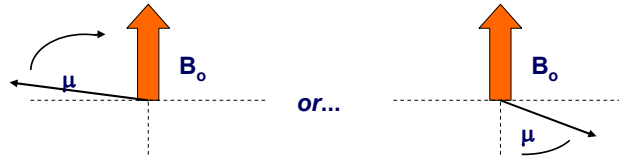
Precession and spinning tops

- What precession is ω_0 associated with? One thing that we left out from the mix is the **angular momentum, I** , which is associated with all nuclei:



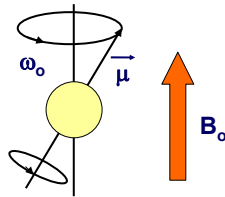
- Crudely, we can think of the nuclei as being spinning around its z axis. If we now consider those nuclei that have also a non zero μ , we have little spinning atomic magnets.

- Now, if we bring about a big B_0 , there will be an interaction between μ and B_0 that generates a torque. No matter which is the original direction of μ , it will tend to align with B_0 :



Precession (continued)

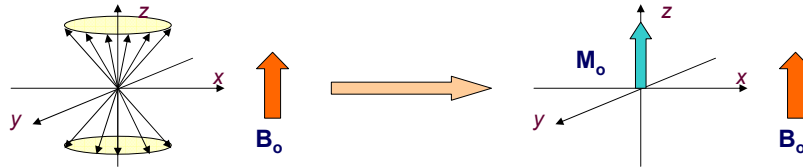
- Since the nuclei associated with μ is spinning due to I , there are two forces acting on it. One that wants to bring it towards B_0 , and one that wants to keep it spinning. μ ends up precessing around B_0 :



- The best way to picture it is to imagine a spinning wooden top under the action of gravity.

Bulk magnetization

- We see the effects on macroscopic magnetization, M_o , which is directly proportional to the population difference ($N_\alpha - N_\beta$), in which contributions from different μ s have been averaged:

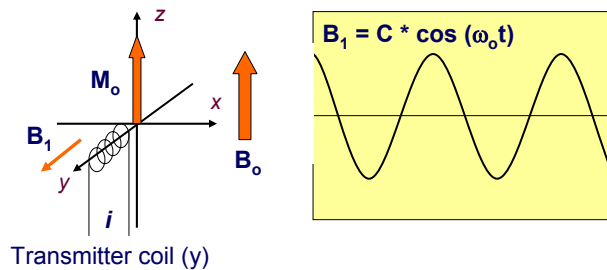


- We can decompose each little μ in a z contribution and an $\langle xy \rangle$ plane contribution. The components in the $\langle xy \rangle$ plane are randomly distributed and cancel out. For the ones in z , we get a net magnetization proportional to $N_\alpha - N_\beta$.
- Since this is (more or less) the situation in a real sample, we will from now on use M_o in all further descriptions/examples.
- There is an important difference between a μ and M_o . While the former is **quantized** and can be only in one of two states (α or β), the latter tells us on the whole spin population. It has a **continuous** number of states.

NMR excitation

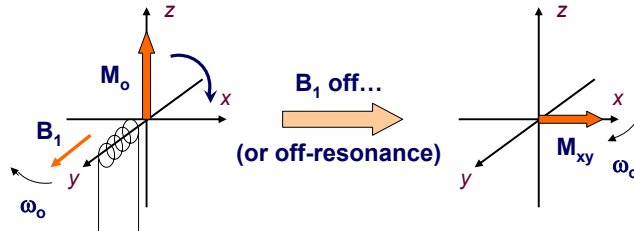
So far, nothing happened. We need to do something to the system to observe any kind of signal. What we do is take it away from this condition and observe how it goes back to equilibrium. This means affecting the populations...

- We need the system to absorb energy. The energy source is an oscillating electromagnetic radiation generated by an alternating current:



Resonance

- When the frequency of the alternating current is ω_0 , we achieve a **resonant condition**. The alternating magnetic field and \mathbf{M}_0 interact, there is a torque generated on \mathbf{M}_0 , and the system absorbs energy :



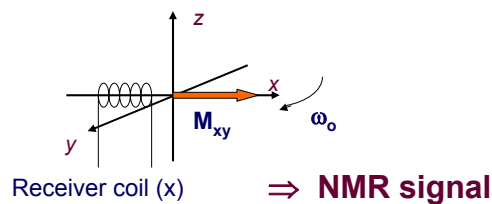
- Since the system absorbed energy, the equilibrium of the system was altered. We modified the populations of the \mathbf{N}_α and \mathbf{N}_β energy levels.
- Again, keep in mind that individual spins flipped up or down (a single **quanta**), but \mathbf{M}_0 can have a continuous variation.

Return of \mathbf{M}_0 to equilibrium (and detection)

- In the absence of the external \mathbf{B}_1 , \mathbf{M}_{xy} will try to go back to \mathbf{M}_0 (equilibrium) by restoring the same $\mathbf{N}_\alpha / \mathbf{N}_\beta$ distribution. This phenomenon is called **relaxation**.
- \mathbf{M}_{xy} returns to the \mathbf{z} axis precessing on the $\langle xy \rangle$ plane:

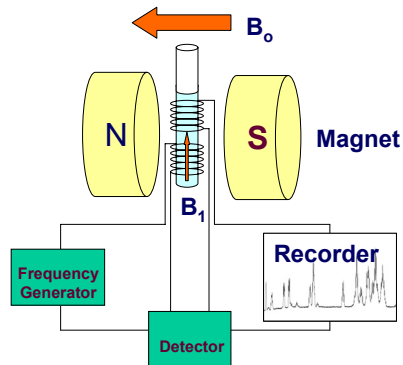


- The oscillation of \mathbf{M}_{xy} generates a fluctuating magnetic field which can be used to generate a current in a coil:



NMR Instrumentation

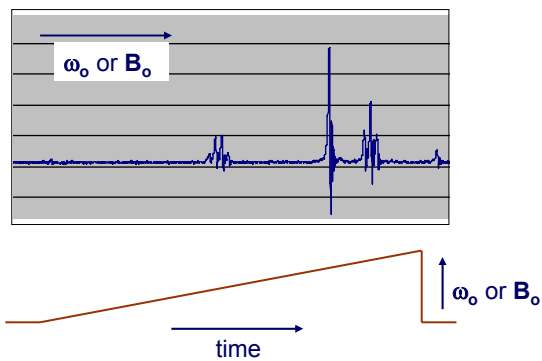
- An NMR machine is basically a big and expensive FM radio.



- **Magnet** - Normally superconducting. Some electromagnets and permanent magnets (EM-360, EM-390) still around.
- **Frequency generator** - Creates the alternating current (at ω_0) that induces B_1 . Continuous wave or pulsed.
- **Detector** - Subtracts the base frequency (a constant frequency very close to ω_0) to the output frequency. It is lower frequency and much easier to deal with.
- **Recorder** - XY plotter, oscilloscope, computer, etc., etc.

Continuous Wave excitation

- It's pretty *de mode*, and is only useful to obtain 1D spectra.
- The idea behind it is the same as in UV. We scan the frequencies continuously (or sweep the magnetic field, which has the same effect - $\omega = \gamma B$), and record successively how the different components of M_0 generate M_{xy} at different frequencies (or magnetic fields).



Fourier Transform - Pulsed excitation

- The way every NMR instrument works today.
- The idea behind it is pretty simple. We have two ways of tuning a piano. One involves going key by key on the keyboard and recording each sound (or frequency). The other, kind of brutal for the piano, is to hit it with a sledge hammer and record all sounds at once.
- We then need something that has all frequencies at once. A short **pulse** of radiofrequency has these characteristics.
- To explain it, we use another black box mathematical tool, the **Fourier transformation**: It is a transformation of information in the time domain to the frequency domain (and *vice versa*).

$$S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt$$

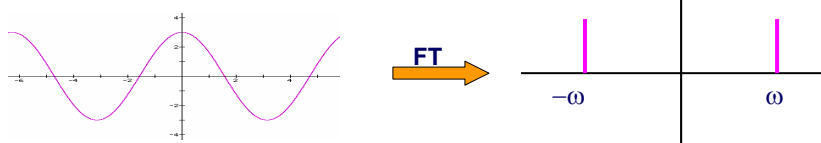
$$s(t) = 1/2 \pi \int_{-\infty}^{\infty} S(\omega) e^{i\omega t} dt$$

- If our data in the time domain is periodical, it basically gives us its frequency components. Extremely useful in NMR, where all the signals are periodical.

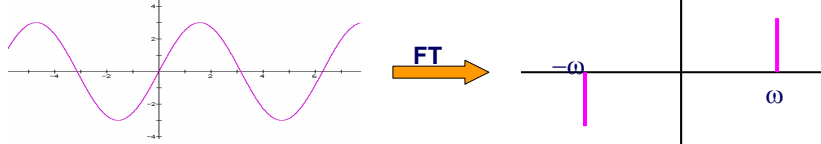
Fourier Transform of simple waves

- We can explain (or see) some properties of the FT with simple mathematical functions:

- For **cos(ω * t)**



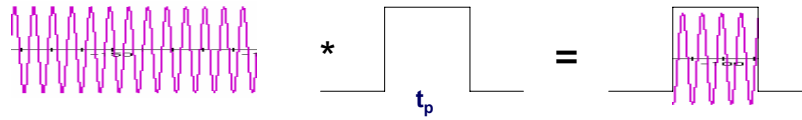
- For **sin(ω * t)**



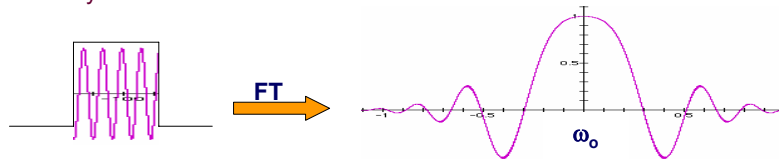
- The cosines are said to give absorptive lines, while sines give dispersive lines.

Back to pulses

- Now that we 'master' the FT, we can see how pulses work. A radiofrequency pulse is a combination of a wave (cosine) of frequency ω_0 and a step function:



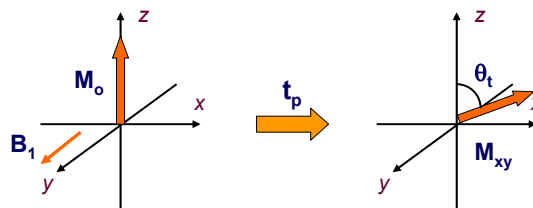
- This is the time domain shape of the pulse. To see the frequencies it really carry, we have to analyze it with FT:



- The result is a signal centered at ω_0 which covers a wide range of frequencies in both directions. Depending on the pulse width we have wider (shorter t_p) or narrower (longer t_p) ranges. Remember that $f \propto 1/t$.

Pulse widths and tip angles

- The pulse width is not only associated with the frequency range (or sweep width), but it also indicates for how long the excitation field B_1 is on. Therefore, it is the time for which we will have a torque acting on the bulk magnetization M_0 :

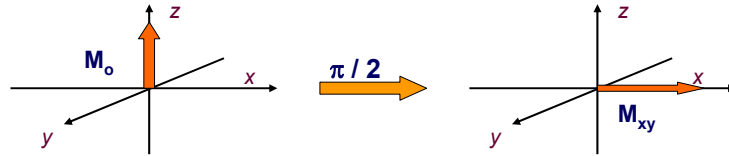


$$\theta_t = \gamma * t_p * B_1$$

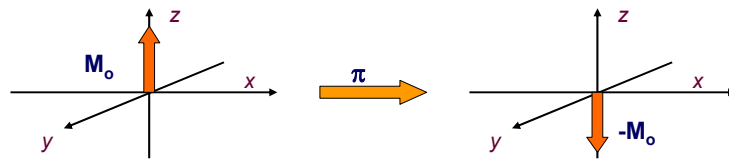
- As the pulse width for a certain flip angle will depend on the instrument (B_1), we will therefore refer to them in terms of the rotation we want to obtain of the magnetization. Thus, we will have $\pi/4$, $\pi/2$, and π pulses.

Some useful pulses

- The most commonly used pulse is the $\pi/2$, because it puts as much magnetization as possible in the $\langle xy \rangle$ plane (more signal can be detected by the instrument):



- Also important is the π pulse, which has the effect of inverting the populations of the spin system...

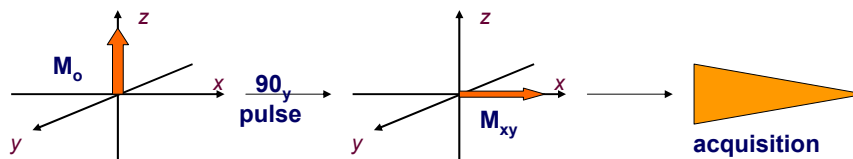


- With control of the spectrometer we can basically obtain any pulse width we want and flip angle we want.

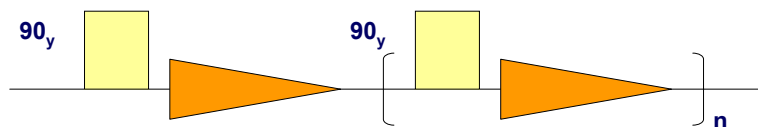
1D Pulse sequences

- We now have most of the tools to understand and start analyzing pulse sequences.

Vectors:



Shorthand:



- According to the direction of the pulse, we'll use 90_x or 90_y (or 90_ϕ if we use other phases) to indicate the relative direction of the B_1 field in the rotating frame.
- The acquisition period will always be represented by an FID for the nucleus under observation (the triangle).

Relaxation phenomena

- So far we haven't said anything about the phenomena that brings the magnetization back to equilibrium. **Relaxation** is what takes care of this. There are two types of relaxation, and both are time-dependent exponential decay processes:

Longitudinal or Spin-Lattice relaxation (T_1):

- It works for the components of magnetization aligned with the **z** axis (M_z).
 - Loss of energy in the system to the surroundings (lattice) as heat.
 - Dipolar coupling to other spins, interaction with paramagnetic particles, etc...

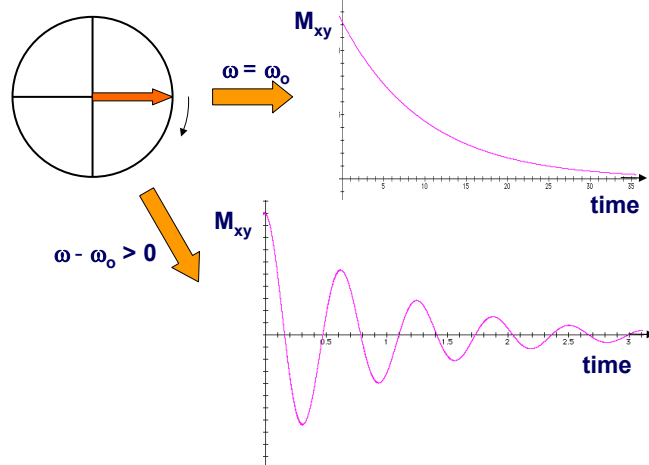
Transverse or Spin-Spin relaxation (T_2):

- It acts on the components of magnetization lying on the **<xy>** plane (M_{xy}).
 - Spin-spin interactions dephase M_{xy}
 - Also by imperfections in the magnet homogeneity (fanning out).
 - Cannot be bigger than T_1 .

Free Induction Decay (FID)

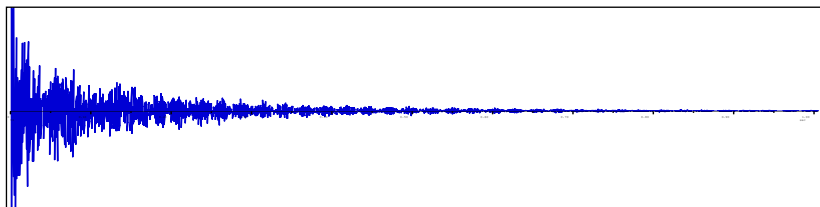
- Now, we are interested in analyzing the signal that appears in the receiver coil after putting the bulk magnetization in the **<xy>** plane ($\pi / 2$ pulse).

- We said earlier that the sample will go back to equilibrium (z) precessing. In the rotating frame, the frequency of this precession is $\omega - \omega_0$. The relaxation of M_0 in the **<xy>** plane is exponential. Therefore, the receiver coil detects a decaying cosinusoidal signal (single spin type):

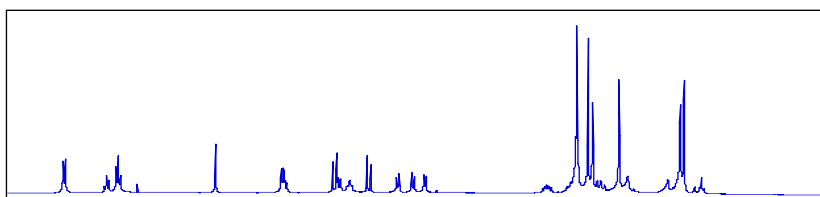


FID (continued)

- In a real sample we have hundreds of spin systems, which all have frequencies different to that of B_1 (or *carrier frequency*). Since we used a pulse and effectively excited all frequencies in our sample at once, we will see a combination of all of them in the receiver coil, called the *Free Induction Decay* (or *FID*):

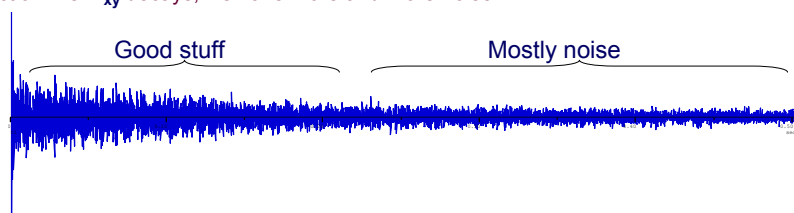


- The FT of this signal gives us the NMR spectrum:

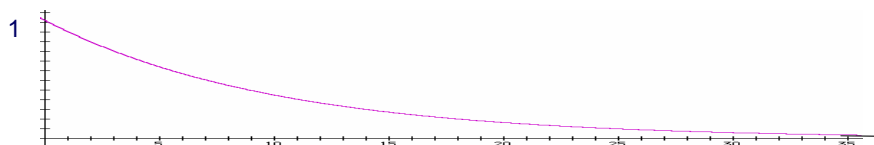


Data processing - Window functions

- Now we have the signal in the computer, properly sampled. There are some things we can do now a lot easier, and one of them is filtering. The real information in the FID is in the first section. As M_{xy} decays, we have more and more noise:



- The noise is generally high frequency, and that is why NMR spectra have this jagged baseline. What if we could filter all the signals that were higher than a certain frequency?
- We use *digital filtering*. Intuitively, it means multiplying the FID by a function that makes the noise at the end smaller:

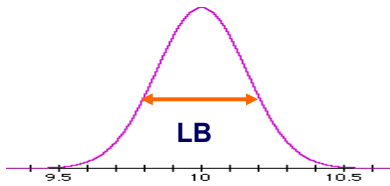


Window functions (continued)

- In this case, it is called **exponential multiplication**, and has the form:

$$F(t) = 1 * e^{-(LB * t)} \quad \text{or} \quad F(t) = 1 * e^{-(t/\tau)}$$

- Why is that this removes high frequency noise? Actually, we are **convoluting** the frequency domain data with the FT of a decaying exponential. The FT of this function is a Lorentzian shaped peak with a width at half-height proportional to the rate of decay, or **line broadening (LB)**, in Hz.



- Convolution makes the contribution of everything with a WAHH thinner than LB smaller in the spectrum (the scale here is bogus...).

- If we use an LB with the opposite sign, the exponential grows instead of decaying, letting signals with narrower widths to pass, improving resolution but lowering signal to noise ratio.

Sensitivity and resolution enhancement

- For the following raw FID, we can apply either a positive or negative LB factor and see the effect after FT:

